**METHOD 2**

Key Assumption: The game is reset every time there is a kickoff (i.e. there is a made field goal or touchdown) and at the end of each half.

This means the game is comprised of two independent halves. And therefore value functions will be defined as expected net points until the end of the half. Furthermore, the analysis of these value functions will be the same regardless of whether the game is in the first or second half.

To convert the value functions defined in terms of expected net points until the end of the half into value functions defined in terms of expected net points until the end of the game is simple. For second half observations they are the same. For first half observations, the expected net points until the end of the game are the expected net points until the end of the half plus-or-minus the expected net points for receiving a kickoff, depending on whether the team with the ball will be receiving or kicking off to start the second half.

To estimate: The 100 values V1, V2, … V100 (assumed to be constant) where

Vx = expected net points until the end of the half for a team that has a first down and 10 or a first down and goal, at a distance of x yards from the team’s own goal line; x = 1, 2, … 99

V100 = expected net points until the end of the half for a team that is set to receive a kickoff

Auxiliary Variables (assumed to be constants):

ENPx = expected net points until either the end of the current half or until the next kickoff (whichever happens first) for a team that has a first down and 10 or a first down and goal, at a distance of x yards from the team’s own goal line. x = 1, 2, … 99

ENP100 = expected net points until either the end of the current half or until the next kickoff (whichever happens first) for a team that is set to receive a kickoff

PrbOffx = the probability that a team that has a first down and 10 or a first down and goal, at a distance of x yards from the team’s own goal line, will make the next kickoff before the end of the current half because either that team is the first team to make a touchdown before the end of the current half or that team is the first team to make a field goal before the end of the current half. x = 1, 2, … 99

PrbOff100 = the probability that a team that is set to receive a kickoff will make the next kickoff before the end of the current half because either that team is the first team to make a touchdown before the end of the current half or that team is the first team to make a field goal before the end of the current half.

PrbDefx = the probability that the opponent of a team that has a first down and 10 or a first down and goal, at a distance of x yards from the team’s own goal line, will make the next kickoff before the end of the current half because either the opponent is the first team to make a touchdown before the end of the current half or the opponent is the first team to make a field goal before the end of the current half. x = 1, 2, … 99

PrbDef100 = the probability that the opponent of the team that is set to receive a kickoff will make the next kickoff before the end of the current half because either the opponent is the first team to make a touchdown before the end of the current half or the opponent the first team to make a field goal before the end of the current half.

Note1: ProbOffx + PrbDefx < 1 because the probability that there will be another kickoff before the end of the current half is less than one; i.e. it is possible that neither team will make a field goal or a touchdown before the end of the current half.

Step 1 – Key equations:

Vx = ENPx – PrbOffx \* V100 + PrbDefx \* V100 x = 1, 2 … 100

Step 2 – Rewrite Key Equations so that V1, V2, … V100 can be estimated by regression. This will allow us to get the correct standard errors of the estimates.

Regression Equation:

Assume the first T1 observations are on first downs and next T2 observations are on kickoffs.

Define

NPt = Net points for observation t, calculated from the start of the first down or prior to receiving the kickoff until the end of the half or the next kickoff, whichever comes first.

Dx,t = 1 if observation t is a first down and 10 or first down and goal,

at a distance of x yards from the team’s own goal line; x = 1, 2, … 99 and

t = 1, 2, … T1

1 if observation t is a kickoff; x=100 and t = T1+1, T1+2, … T2

kj

0 otherwise

## OFFx,t = 1 if on observation t the first down is x yards from the offensive team’s own goal line and the offensive team is the first team to make a field goal or a touchdown before the end of the half , x = 1, 2, … 99 and t = 1, 2, … T1

1 if on observation t the team receiving the kickoff is the first team to make a field goal or a touchdown before the end of the half , x = 100 and

t = T1+1, T1+2, … T2

0 otherwise

DEFx,t = -1 if on observation t the first down is x yards from the offensive team’s own goal line and the defensive team is the first team to make a field goal or a touchdown before the end of the half , x = 1, 2, … 99 and t = 1, 2, … T1

-1 if on observation t the team kicking off is the first team to make a field goal or a touchdown before the end of the half, x = 100 and

t = T1+1, T1+2, … T2

0 otherwise

WHOx,t = OFFx,t + DEFx,t ; x = 1, 2, … 99 and t = 1, 2, … T1

1 + OFFx,t + DEFx,t ; x = 100 and t = T1+1, T1+2, … T2

Then

where

Note that V1, … V100 are the regression coefficients in equation (1).

Estimation technique 1:

Dx is uncorrelated with ε for x = 1, … 100

Math –

Intuition – Dx measures which type of observation (e.g. 1st down 37 yards from own goal, 1st down 95 yards from own goal, or a kickoff). The residual measures the deviation of the outcomes of net points from what was expected plus the deviation of the outcomes of who made a field goal or touchdown first from what was expected. The correlation between Dx and ε is thus the average of these deviations over samples that will cause them to average out to zero.

WHO is correlated ε

Math

Intuition – WHO is a realization of what actually happened. What actually happened is correlated with the deviation of the outcome from its expectation.

D100 is correlated with WHO

Math

Intuition –

So, estimate (1) using instrumental variables; instruments are Dx, x=1,…100

To get the correct standard errors for the estimates of V1

Cluster standard errors by half.

Estimation Technique 2;